Internal Waves with Rotation: Wall Localisation and Wakes

Fiacra McCluskey, Marek Stastna, University of Waterloo

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Video of wave available here

Introduction

In the study of internal waves, the effects of rotation are often ignored due to their complex three dimensional nature. x, the direction of wave propagation, and z, the vertical direction of density stratification, are necessary to all internal waves. However the spanwise direction, y, can be ignored when initial conditions, and the domain are both spanwise invarient.

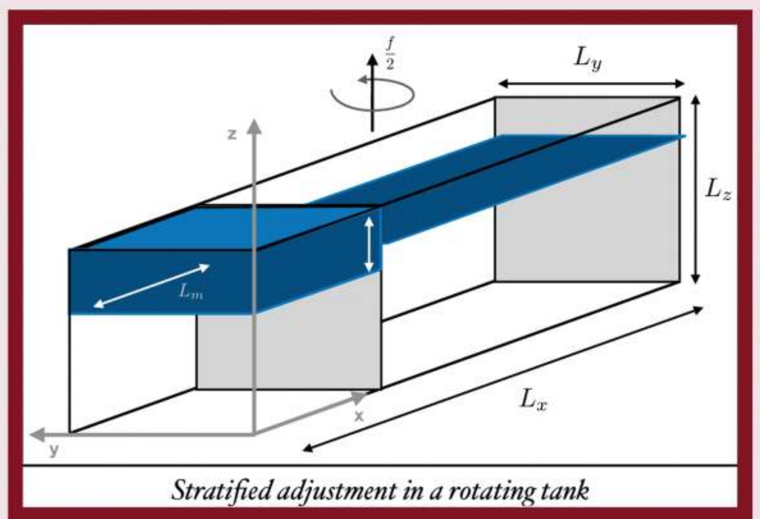

Figure 1. Experimental setup of the rotating tank

Making the Boussinesq and f-plane approximations, rotation is reduced to accelerations of f*v in the x direction and -f*v in the y direction, thus, a wave moving in the positive x direction is deflected towards negative y. In our setup. The Rossby number or our system, which measures strength of rotation, is u/fL = 1.3. This means rotational and non rotational forces have similar strengths.

As seen in Figure 1, we release a mode 1 wave of light water in an long shallow tank (Lx=65.5, Ly=1.5, Lz=0.25). Intensive numerical simulations were undergone to analyze the effects of rotation on such a wave with 4096 by 384 by 128 points. Without rotation we expect a wave train of solitary waves (larger amplitude waves move faster) to form (Stastna, 2001). With rotation all waves in our box will curve into the bottom (y=0) wall due to effects described above. Energy is focused on that wall, similarly to seen in the analytically described Kelvin wave solution to the rotating wave problem along a single coastline, where v=0 is assumed and amplitude drops off in y (Kundu, 1990, p 509).

3D Visualization of Solitary Wave and Close Wave Trail

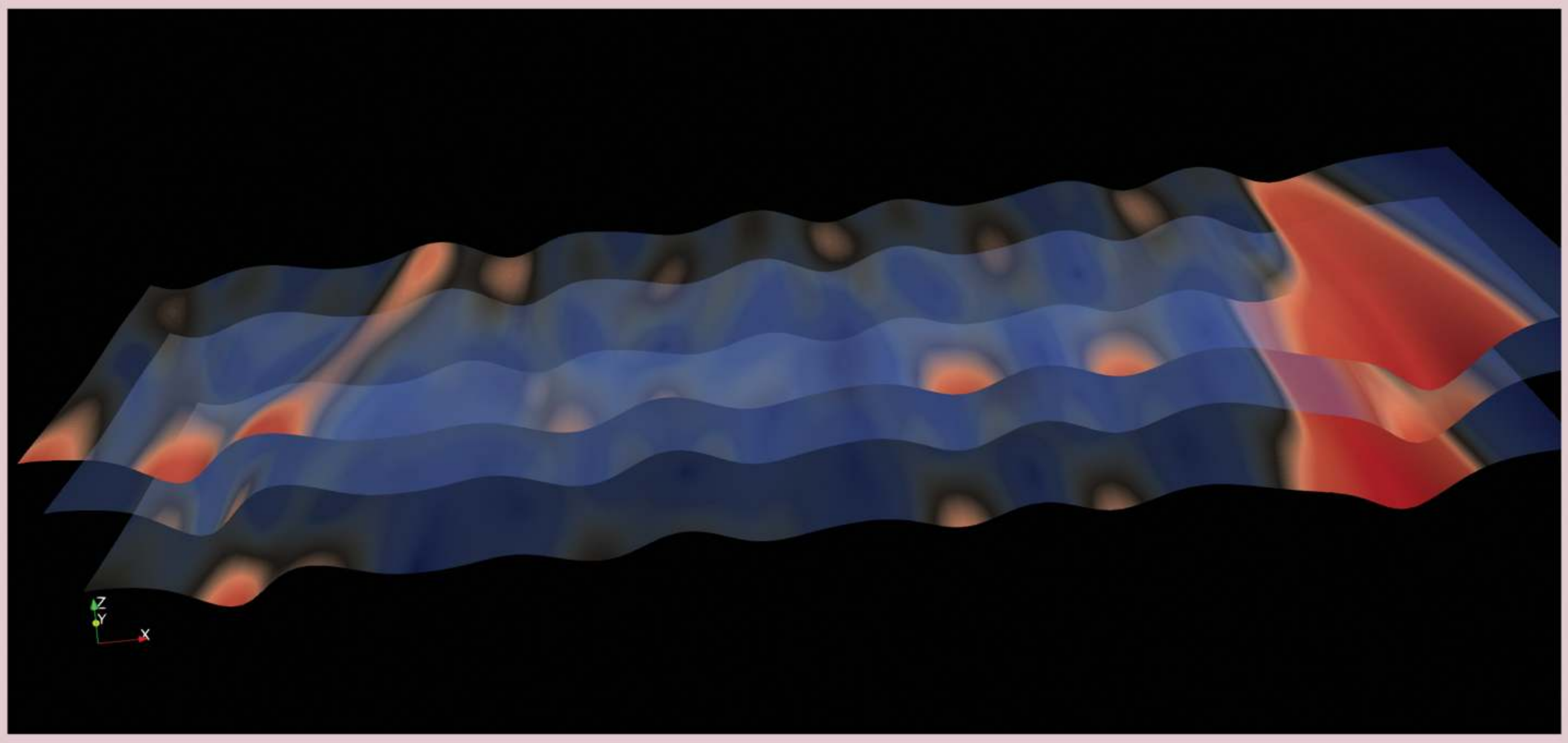

Figure 2. Three constant density surfaces at the final timestep, coloured by Kinetic Energy (log scale). Figure shows focusing of the main wave to the bottom wall in both Kinetic energy and density depression. Kinetic Energy is low in the vertical center due to the wave being mode 1 – u is positive on the top slice and negative on the bottom slice. Wave trail includes several smaller disturbances following the main wave on the top and bottom walls where Poincare waves meet, and are transported backwards along Poincare waves over time, this can be seen occurring at the back of the visible wave trail. Wave has progressed for ~40 wavelength.

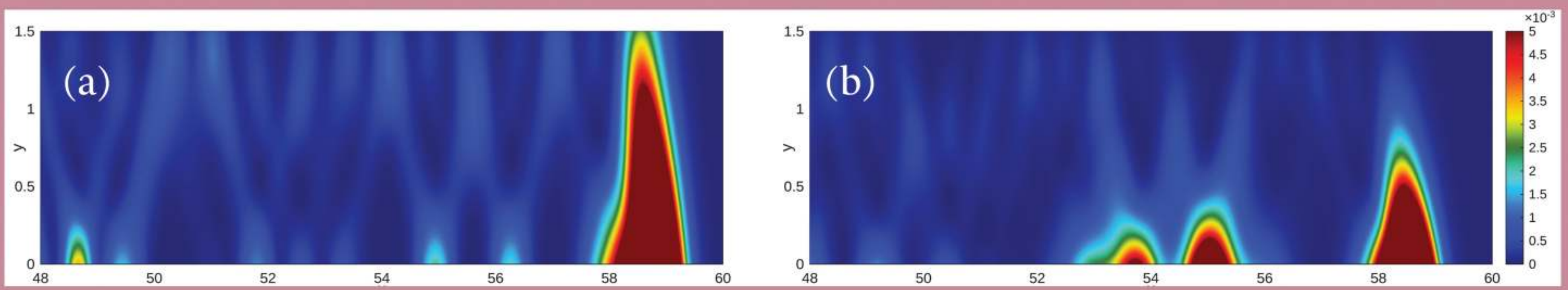

Figure 8. Kinetic Energy at the final timestep for (a) default wave, (b) wave with double rotation rate. The change in rotation rate causes energy to be even more focused on the close wall, and trailing waves to be far larger. Rather than bouncing between walls, the wave trail energy also remains largely fixed on the close wall. Poincare waves are less prominent. The final wave more closely resembles a Kelvin wave with an exponential decay of amplitude in the y direction and never developing the disticive “shelf”.

Figures 3-7 shows a 3D visualization of the wave through Kinetic Energy. View follows the wave with a speed of around x/t = 1.6. Wave Potential Energy closely follows Kinetic and is of comparable size. Energy over entire domain is slowly lost at a constant rate. Clear regions represent very low Kinetic energy.

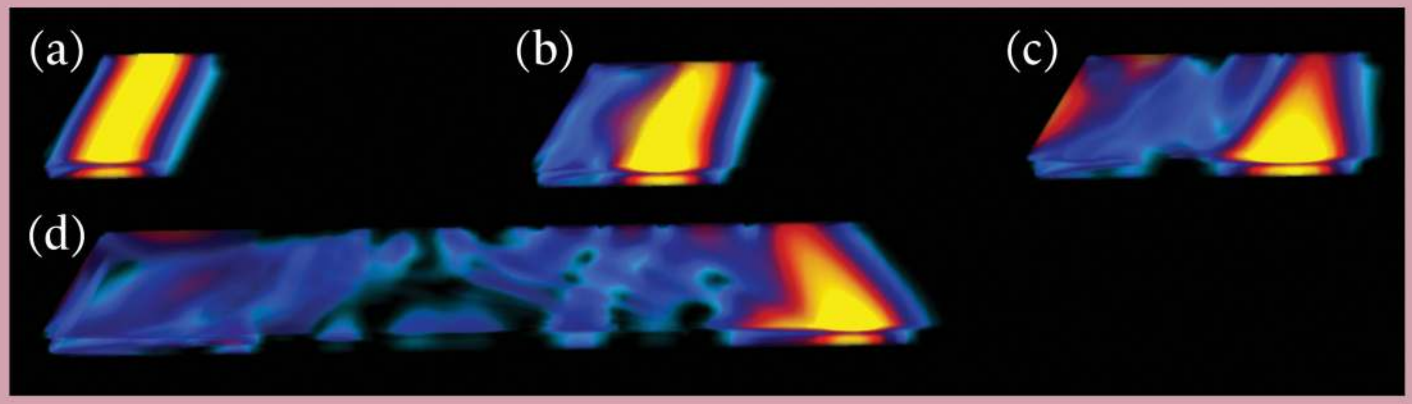

Figure 3. Early stages of the wave at (a) t=1, (b) t=2, (c) t=4, (d) t=8. Energy focuses on the bottom wall and the wave curves backwards. Distinct ‘shelf’ forms behind the wave on the bottom wall and will contunie to exist in figures 4-7, although diminished in 7.

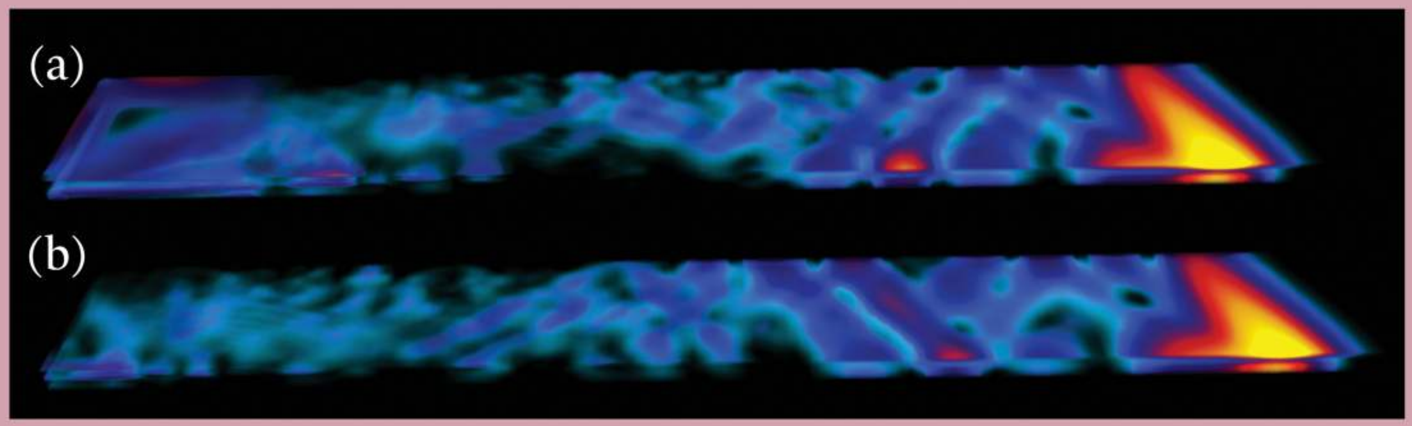

Figure 4. Wave at (a) t=16, (b) t=24. Focusing effect of Poincare waves (gravity waves) in the trail (a) appears at bottom wall, (b) is in transition from bottom wall to top wall. This focal point of energy in the trail moves backwards relative to the leading wave.

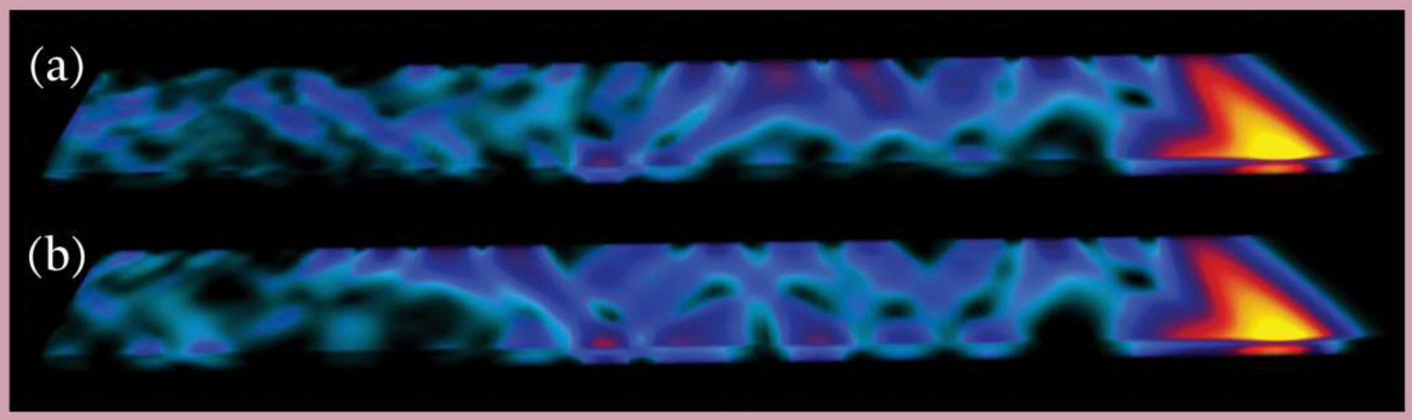

Figure 5. Wave at (a) t=30, (b) t=40. Wave trail energy maxima due to focusing continues to bounce between walls while moving backwards in the wave trail compared to where it was visible in figure 4. Leading wave appears very steady in time.

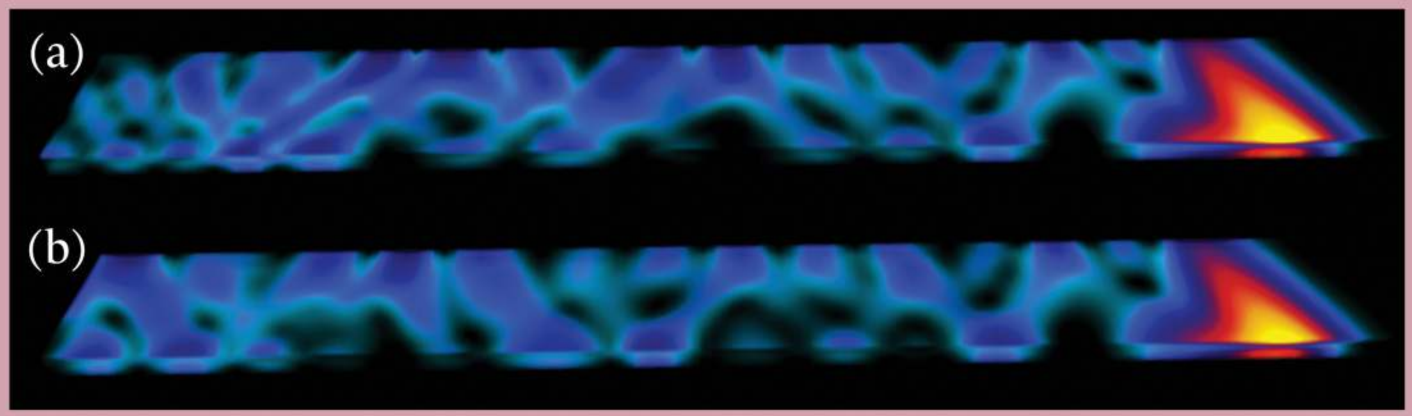

Figure 6. Wave at (a) t=50, (b) t=65. Wave trail energy has decreased and appears sparse, particularly close behind the leading wave. ‘Shelf’ has decreased in size as leading wave looses amplitude due to effects of friction, but overall shape is unchamged.

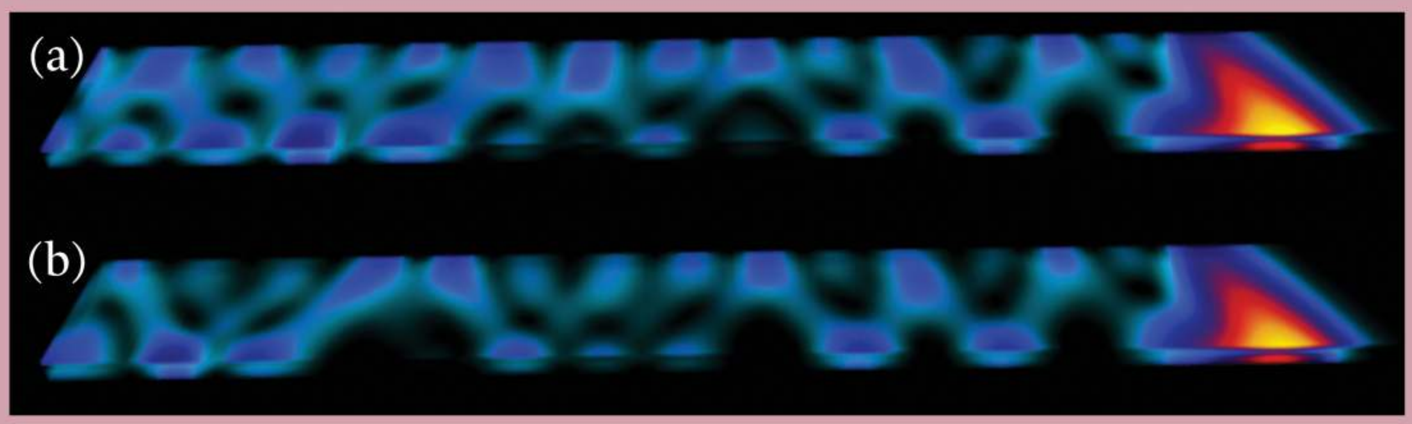

Figure 7. Final wave of the simulation at (a) t=80, (b) t=103. Amplitude of main wave has visibly decreased. Close wave trail is steady but further behind leading wave, the trail is still changing. Wave has traveled ~40 wavelengths by (b) (same t value as figure 2).

References

We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC).

Kundu, P. K. Fluid Mechanics. Academic Press, San Diego, 1990
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Deepwell, D., Stastna, M., & Coutino, A. (2018). Multi-scale phenomena of rotation-modified mode-2 internal waves. Nonlinear Processes in Geophysics, 25, 217–231.