

25

Oceananigans ensemble members

$97 \times 385 \times 33 \times 121 = 149,118,585$

grid points per ensemble member

$R^2 = 0.605$

our HM24-style surface-only CNN

$R^2 = 0.73$

HM24 surface-only CNN benchmark

1. Motivation and HM24 context

Scientific target. Infer subsurface vertical velocity from remotely observable surface fields:

$$\mathcal{F}_\theta : (u_{\text{surf}}, v_{\text{surf}}, b_{\text{surf}}) \rightarrow w(x, y, z).$$

Why this matters. Submesoscale w controls vertical exchange of heat, nutrients, and carbon, but it is difficult to observe directly. **He & Mahadevan (GRL 2024) reference problem.**

- Idealized coastal-upwelling simulations in a 500 m deep, 96×384 km re-entrant channel.
- HM24 compared MLR, RF, and CNN approaches for diagnosing w from surface data.
- Their CNN used a 32×32 km surface patch (with unit stride) and predicted a vertical profile $w(z)$ below the patch center.

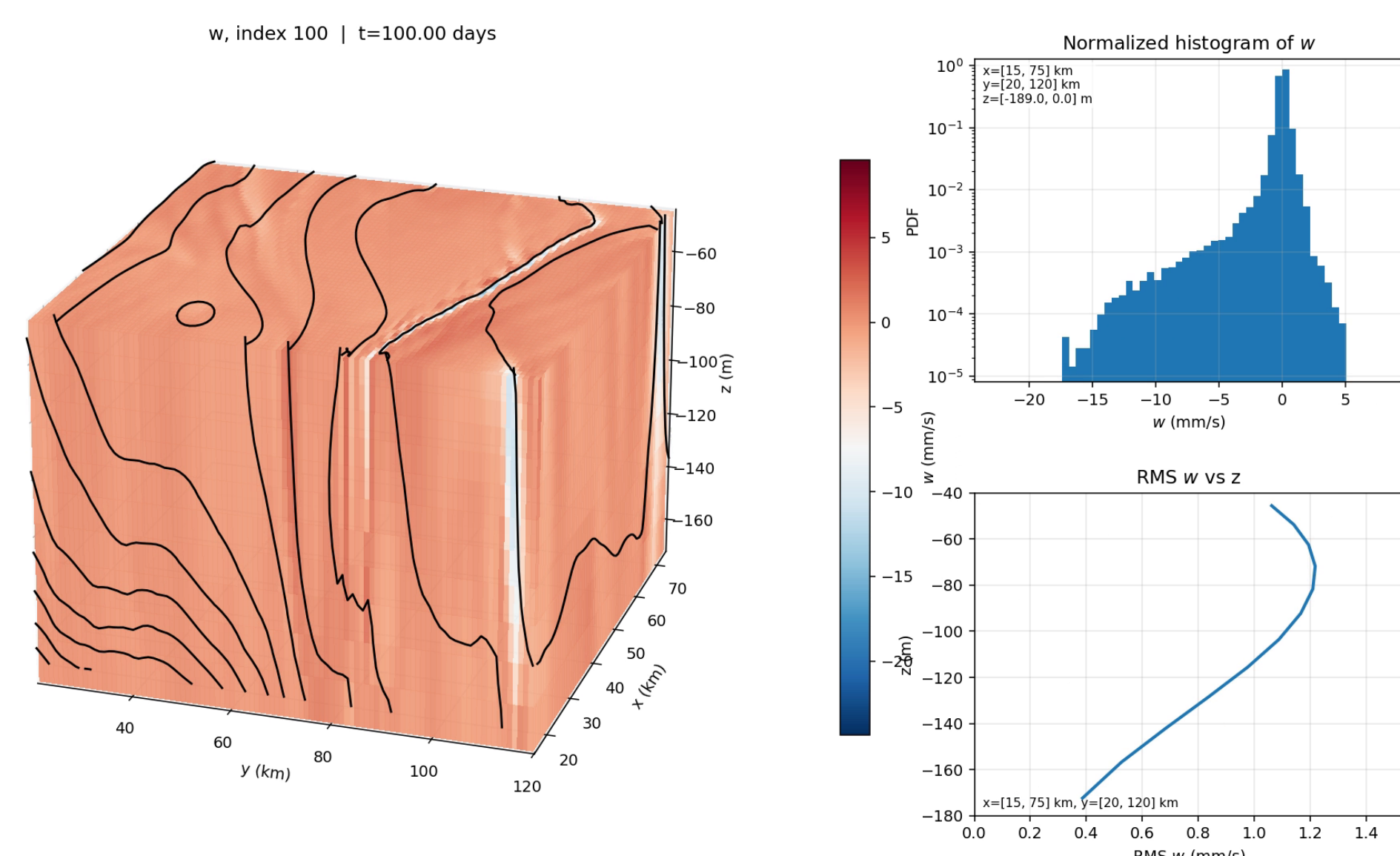
2. Oceananigans simulation design

HM24 implementation

- The model follows from a previous work (He & Mahadevan (JGR Oceans 2021)).
- They use a PSOM governed by 3-D Nonhydrostatic Boussinesq equations.

Reimplementation choices.

- Nonhydrostatic Boussinesq model, WENO-5 advection, RK3 time stepping in Oceananigans.
- f -plane at 15°N , alongshore wind stress ramped over 10 days.
- Horizontal spacing: 1 km; 32 stretched vertical levels over 500 m with nonuniform heights.
- b -diffusivity uses $K_h = 1 \text{ m}^2 \text{ s}^{-1}$ plus wind-dependent $K_z(z)$.
- Small deterministic v perturbation breaks exact alongshore symmetry.



HM24-like model w diagnostic diagrams from ensemble member 19.

Expanded parameters. We replace HM24's 3×3 sweep with a 5×5 sweep:

$$\tau = [0.01, 0.0325, 0.055, 0.0775, 0.1] \text{ N m}^{-2},$$

$$N^2 = [1, 3.25, 5.5, 7.75, 10] \times 10^{-5} \text{ s}^{-2}.$$

3. C-grid to ML grid

The training scripts collocate predictors and targets on tracer-cell centers:

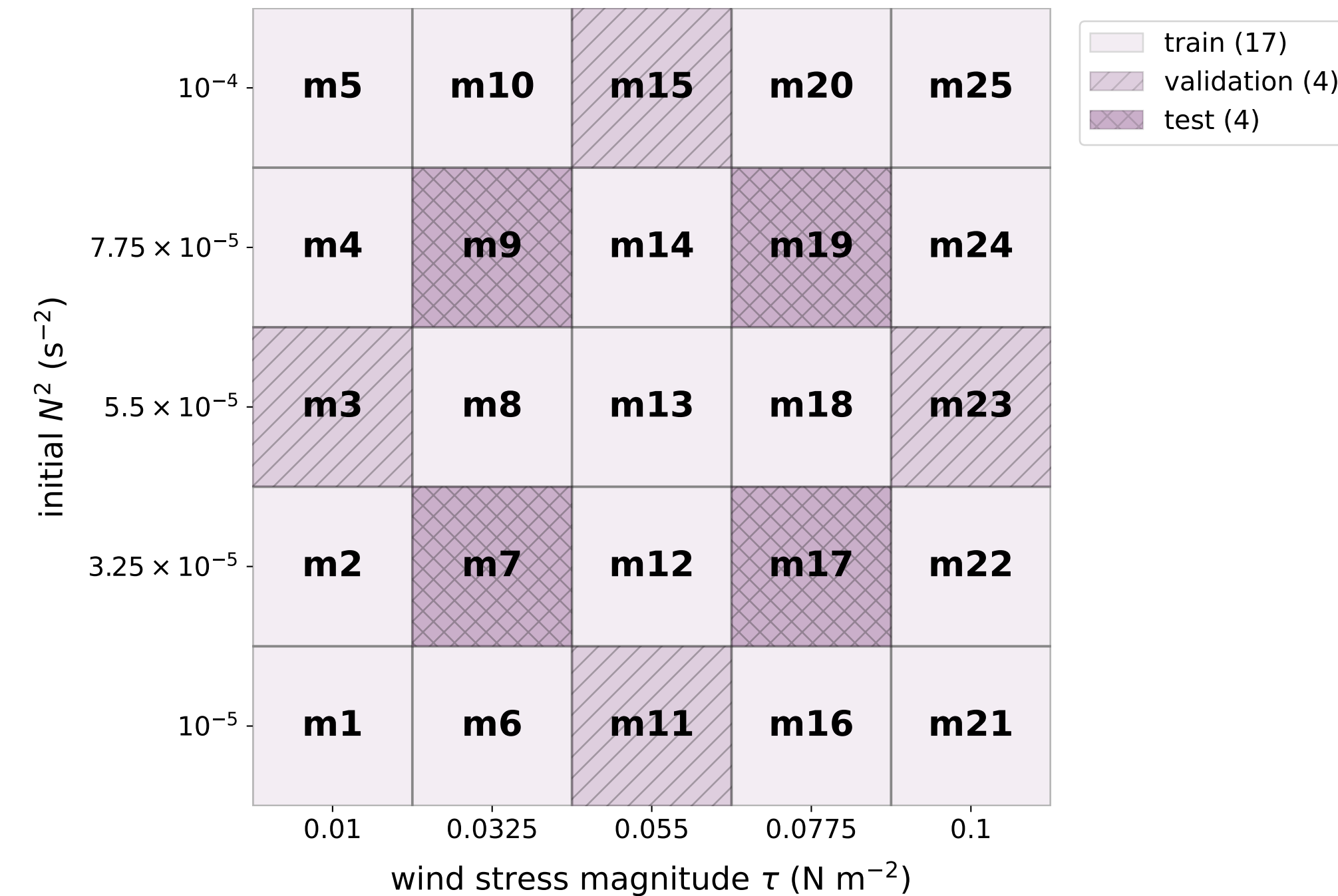
- b already center/center/center; use the uppermost center level.
- u average adjacent x-faces to x centers with periodic wrapping.
- v average adjacent y-faces to y centers.
- w linearly interpolate z-faces to tracer z centers.

For slice models, the target $z = -200$ m maps to the nearest tracer-center level, $z \approx -197.85$ m.

4. 25-member ensemble split

For our own U-Net and CNN runs, we choose a train / validation / test-split based on parameters (ensemble members).

25-member Oceananigans ensemble and member split for slice models



Each cell is one Oceananigans run. Slice models use a member split: train on 17 members, validate on 4, test on 4.

For the HM24-style CNN, we mimic their train / test-split by choosing a 20-day period from each ensemble member, using 4 days for training and 1 day for testing with adjacent dates separated by 5 days.

5. Training methods

Current archive: neural models on Oceananigans data.

- **U-Net slice model:** predicts a full horizontal w map at one target depth from surface fields.
- **Plain CNN slice model:** same target and inputs, but a simpler residual fully convolutional baseline.
- **HM24-style patch CNN:** predicts 21 depths from 0 to -200 m at the center of a local surface patch.

Training emphasis

- Surface-only inputs for current results: $(u_{\text{surf}}, v_{\text{surf}}, b_{\text{surf}})$.
- Member-split evaluation tests generalization to unseen wind/stratification combinations.
- Staggered C-grid variables are averaged/interpolated before training.

6. Data splits and comparability

Slice models: harder member split.

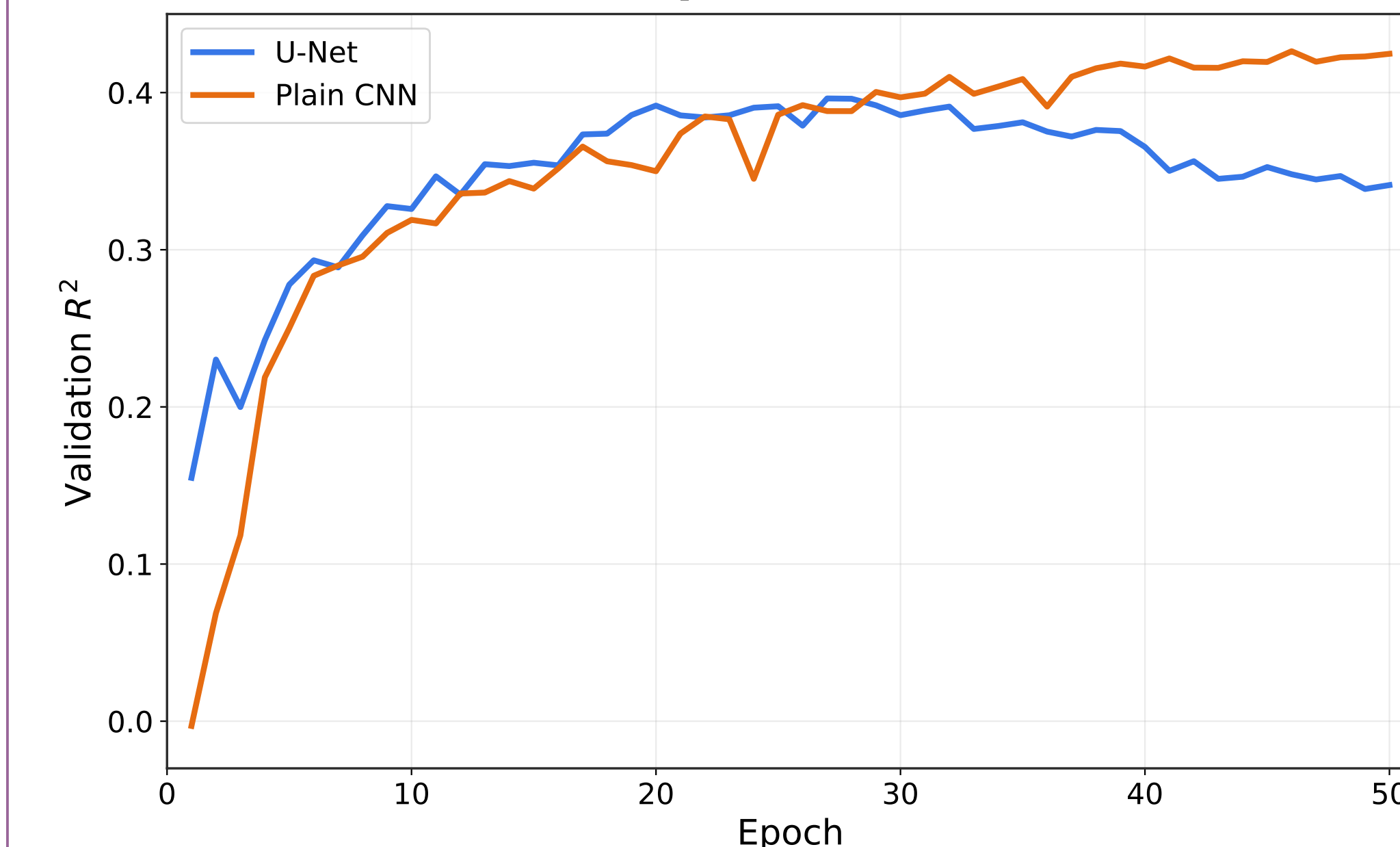
- Train/validation/test members are disjoint.
- The task is to generalize to unseen parameter combinations.
- The predicted target is one deep upper-ocean slice over the full domain.

Patch CNN: closer to HM24.

- Uses all 25 members, but different output times for train and test.
- Predicts a center-column profile from local 32×32 surface patches.
- Evaluation is upper-200 m aggregate skill, matching HM24 more closely.

7. Training progress

Member-split validation skill



Plain CNN slightly outperforms U-Net on the member-split slice task; improvement slows after about 20–30 epochs.

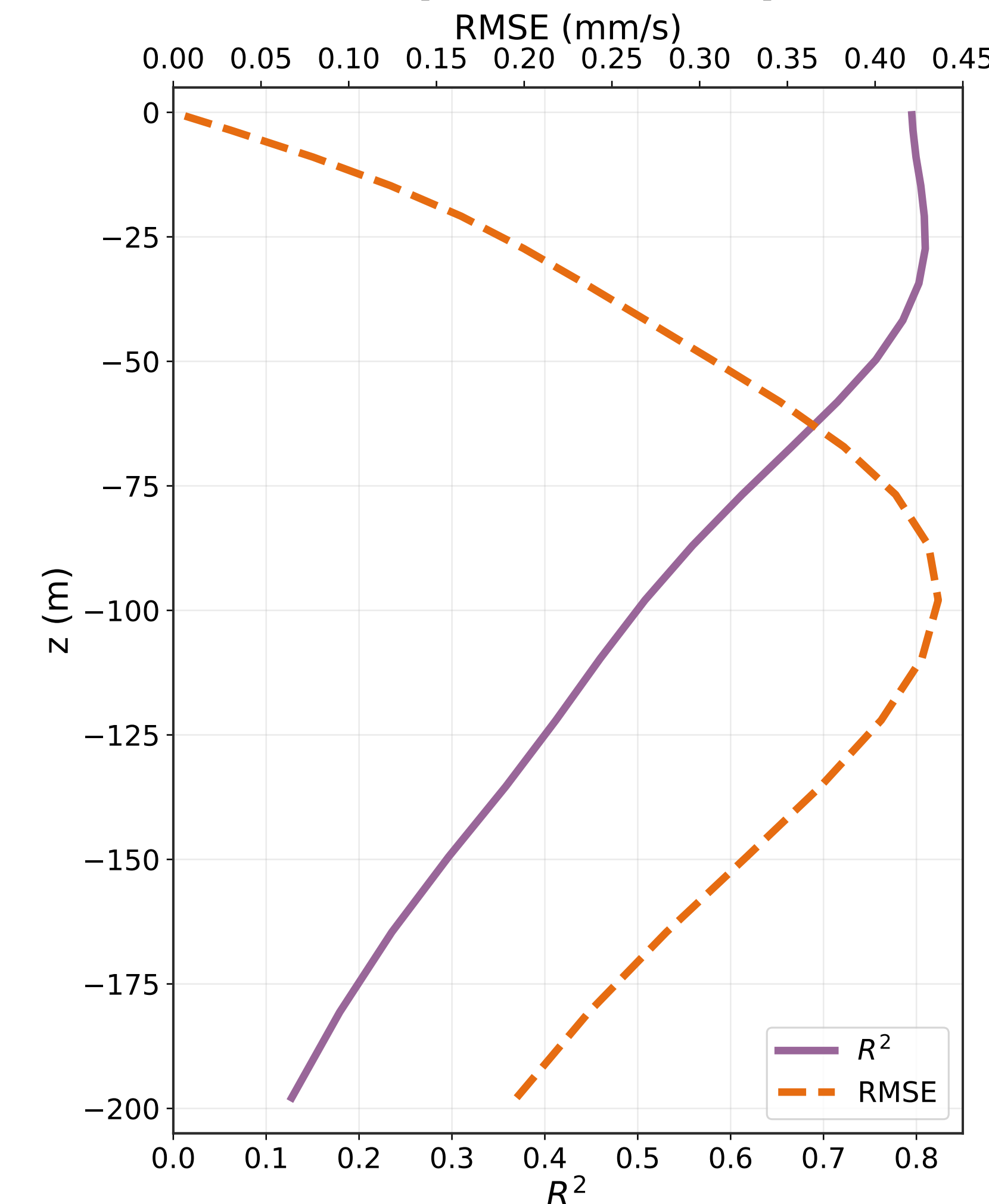
8. Metric definitions

$$R^2 = 1 - \frac{\sum_i (w_i - \hat{w}_i)^2}{\sum_i (w_i - \bar{w})^2}, \quad \text{RMSE} = \sqrt{\langle (\hat{w} - w)^2 \rangle}.$$

HM24's table reports an upper-200 m aggregate R^2 over the test set, not a simple average of depthwise R^2 values. We therefore compare both aggregate and depth-resolved skill when possible.

9. Depth profile skill

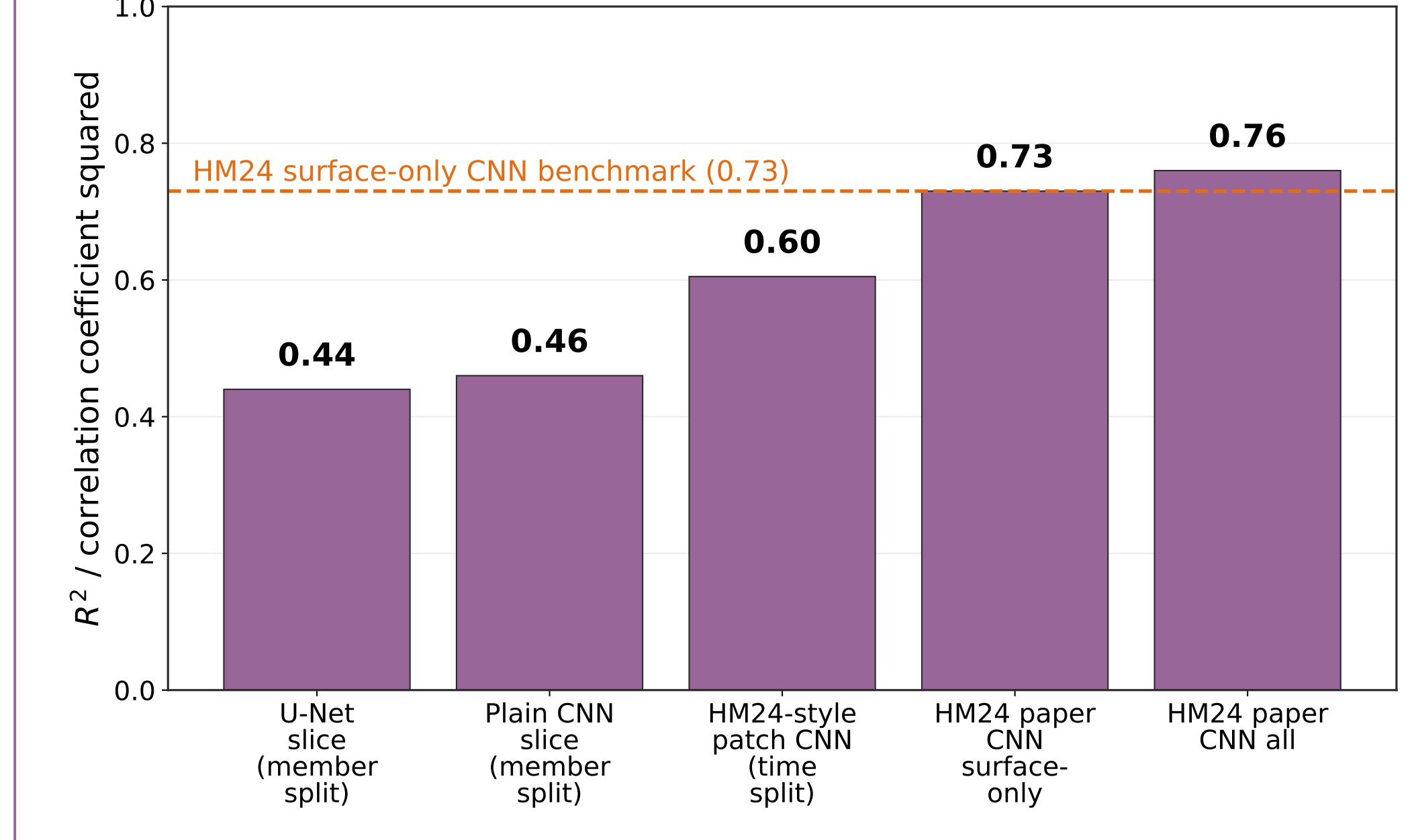
HM24-style surface-only CNN vertical performance profile



The HM24-style CNN is strongest near the surface and loses skill with depth, consistent with surface fields being more directly connected to mixed-layer w .

10. Current results

Current ML skill relative to HM24 benchmarks



Model/test setting	Skill
U-Net slice, member split	$R^2 \approx 0.44$
Plain CNN slice, member split	$R^2 \approx 0.46$
HM24-style patch CNN, time split	$R^2 \approx 0.60$
HM24 paper CNN surface-only	$R^2 \approx 0.73$
HM24 paper CNN all inputs	$R^2 \approx 0.76$

11. Interpretation

- Current member-split results are lower than HM24's time-split CNN benchmarks.
- Part of the gap is expected: member-split testing is stricter, target design differs, and our data come from an Oceananigans reimplementation rather than HM24's PSOM output.
- The patch CNN is closest to HM24 because it predicts a center-column vertical profile instead of a full 2D map.
- Skill is strongest near the surface and decreases with depth.

12. Why are we below HM24?

- Stricter generalization:** member split versus time split.
- Different target:** full 2D deep slice versus center-column profile.
- Expanded ensemble:** 25 members increase variability and challenge.
- Input sensitivity:** surface divergence, stratification effects, and coarsening choices can change skill.

13. Next steps

- Reproduce HM24's train / test time split and patch-CNN setup.
- Move to PINNs for better performance.
- Try iterative and GNN-like architectures.

14. Acknowledgements

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