Is there a slow-fast cross-talk in equatorial turbulence?

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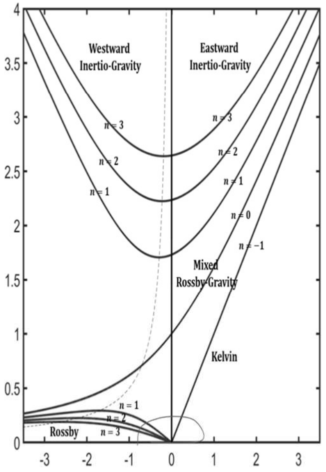
Background

- At midlatitudes, *f*-plane theory indicates that ocean dynamics is a linear superposition of a steady geostrophic field (ω=0) and a fast-evolving inertiogravity wave field (ω<*f*). Modified by nonlinearity and forcing, the steady mode does evolve, but slowly.
- Allowing for beta such that *beta L << f*, we find that the PV gradient largely affects the slow mode, as *iω q' = beta v'* requires vanishing *q'* for the fast mode. This means that Rossby dynamics (westward phase propagation) *only* affect the geostrophic modes. Conversely, only the *f*-dependent IGWs are meridionally trapped.
- The twin constraints of timescale-separation and PV conservation ensure that the Rossby and IGW modes belong to separate turbulent cascades: respectively, inverse (balanced by friction) and forward (balanced by viscosity)...at *mid-latitudes*

Motivation

- At the equator, geostrophy fails (especially zonally). As *f* -> 0, the frequency gap disappears. *Unbalanced (pressure-driven) slow motion can now exist at the equator*. The faster eigenmodes pick up weak PV dynamics, and other transitional behaviours appear, bridging the frequency gap.
- The explicit y-dependence of the coefficients forces *all* modes to be zonally trapped. Consequently modes that extend far off the equator inherit geostrophy and a strong frequency gap, but the grave modes are ageostrophic (at least zonally). Note that higher baroclinic modes have a shorter meridional scale.
- Equatorial mode theory is frequently invoked to explain the equatorial undercurrent, deep jets, and ENSO. "Bridging turbulence", if it exists, may modify our understanding of these dynamics, as well as of the ocean energy budget.



Meridional Modes

- Reducing our equations to a single variable (*v*) we obtain:

$$\tilde{v}''(y) - \left(\frac{\beta y}{c}\right)^2 \tilde{v}(y) = \left[k^2 + \frac{\beta k}{\omega} - \left(\frac{\omega}{c}\right)^2\right]\tilde{v}(y)$$

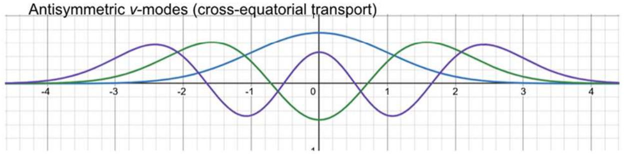
  where $f = \beta y$

- Which is an eigenvalue equation invariant under a meridional Fourier transform: $\tilde{v}''(Y) - Y^2\tilde{v}(y) = \lambda\tilde{v}(y) \Leftrightarrow -l^2\hat{v}(l) + \hat{v}''(l) = \mu\hat{v}(l)$

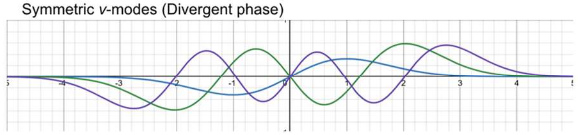
  and we thus seek cousins of the standard Gaussian, which are the *Hermite functions*. Structures for *v* diagrammed below.

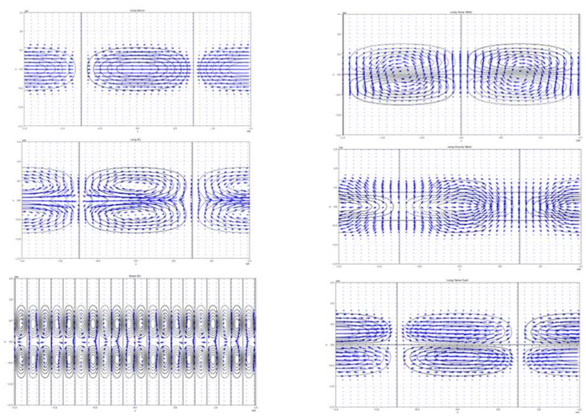
- Note that the explicit *f*-dependence means even the Rossby modes are meridionally trapped. For low *n*, one may ask if the vortical and IGW modes overlap enough to allow turbulent transfers.

Antisymmetric *v*-modes (cross-equatorial transport)



Symmetric *v*-modes (Divergent phase)





- The long Rossby waves and Kelvin waves are all *nondispersive*, have sufficient group velocity to cross an ocean basin, and are low frequency. These are meridionally geostrophic–a common filtering scheme. Note that Kelvin waves have no PV.
- Shorter Kelvin waves and very short gravity waves can also resonate with long Kelvin waves! This is one route for slow-to-fast turbulent leakage.
- The antisymmetric Yanai mode transitions from gravity-like to Rossby-like. Note that group propagation is always eastward, and that MG neglects this wave. Such a wave can interact with another anytisymmetric wave to project onto symmetric modes (or vice-versa). It also is produced upon reflection of MG antisymmetric modes at a Westward boundary.
- Short Rossby waves dominate the famously nonlinear Western Boundary Current.

Methodology (Ongoing project)

- Shallow Water Model on a zonally periodic equatorial beta-plane
- Develop subroutines to initialise specific modes and to project output fields onto these modes
- Run simulations with initial conditions involving:
  - Only Rossby modes (plus some noise with intermediate modes filtered)
  - Kelvin modes as well
  - Yanai modes
- And project the output at a later time onto these modes to see if the spectrum $E(\omega)$ shows a leak of energy to slow modes. Also see whether energy leaks away from the MG modes.

Future directions

- Idealised:
  - Add zonal boundaries, to include coastally trapped waves and reflections
  - Impose symmetric and antisymmetric forcing
  - Vary the base state to incorporate latitudinal shear (symmetric and antisymmetric). Note that an antisymmetric base state introduces inertial instability and may trigger a forward cascade of the Kelvin wave
- Multilayer/continuously stratified models (MIT-GCM). We will repeat a similar sequence of experiments to measure turbulent cascades through the intermediate modes at the equator.

References

- An, S.-I., C. Wang, and C. R. Mechoso, 2020: Teleconnections in the atmosphere. Interacting Climates of Ocean Basins: Observations, Mechanisms, Predictability, and Impacts, C. R. Mechoso, Eds., Cambridge University Press, 54-88

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