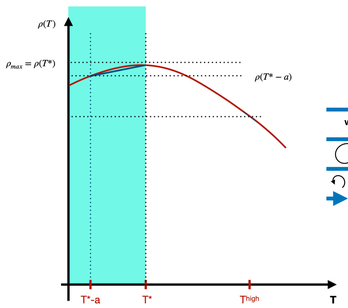
Stratified Shear Instabilities in Cold Water

Khush Bhavsar, University of Waterloo

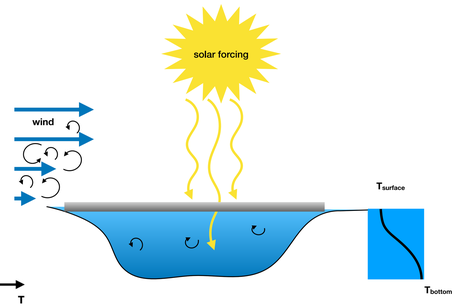
Cold Water Dynamics

- In Canadian winters, most small-medium sized lakes, completely freeze over with ice and snow on top.
- Additionally, all of the water is between 0-4°C, with an *inverse thermal stratification*, i.e., colder (less dense) water (less dense) is on top of warmer (denser) water due to a monotonically *increasing* nonlinear equation of state (EOS).

Nonlinear EOS



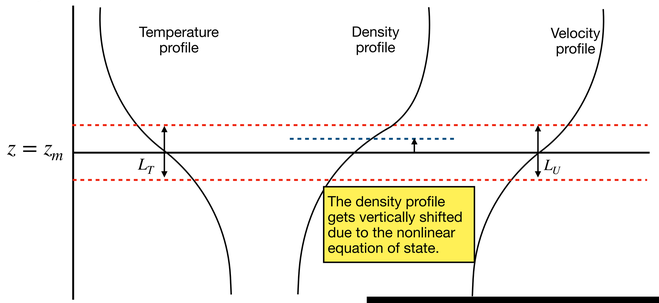
Winter Lakes



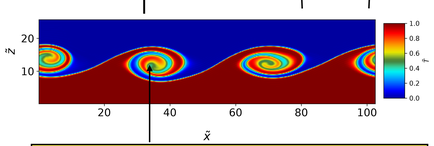
- Under these conditions, wind-input is blocked and thermal input due to solar radiation is reduced. Under-ice shear flows leading to stratified shear instabilities govern dynamics.
- There are two main types of instabilities that occur: the **Kelvin-Helmholtz Instability** (KHI) and the **Holmboe Instability** (HI). We study these using direct numerical simulations (DNS) in a lab-scale tank.

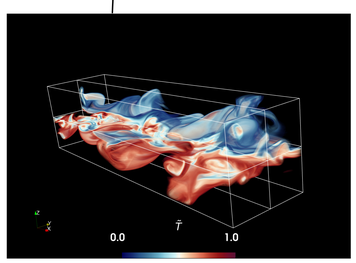
Previous Work: Kelvin-Helmholtz Instability

- When the thickness of the thermocline (region of high temperature gradient) and the shear layer is the same, the instability leads to billows (shown below) which eventually pair up and lead to secondary instabilities and three-dimensional flow.
- An important caveat in the cold water regime is that, since the pycnocline (region of high density gradient) shifts vertically due to the nonlinear EOS, the resulting state after the instability.



The density profile gets vertically shifted due to the nonlinear equation of state.



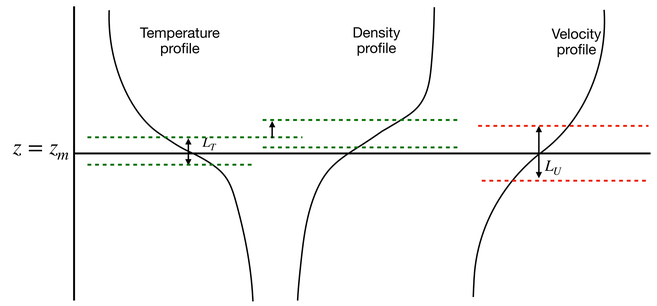


A KHI is characterized by billows that completely overturn along the density interface. This results in sharp interfaces within the core where fluid gets mixed due to thermal diffusion. These billows then pair up and lead to secondary three-dimensional instabilities, eventually turning turbulent.

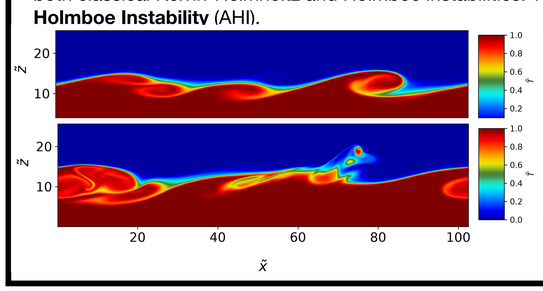
- We discuss comparisons between the true nonlinear equation of state and the linearized one in the cold water regime, along with the effect of Prandtl number and a discussion on the three-dimensional state of the instability in [1] and [2.]

The Holmboe Instability

- If the thermocline is narrower $(L_U/L_T > 2)$ than the shear layer, we can get instabilities with oscillations propagating in opposite directions across the center (z_m). This is called the **Holmboe Instability**.



- Due to a combination of the vertical shifting of the pynocline and narrowing of the thermocline (and therefore, the pycnocline), we observe a mix of characteristics seen in both classical Kelvin-Helmholtz and Holmboe instabilities. This is called an **asymmetric Holmboe Instability** (AHI).



An AHI is characterized by the formation of a cusped primary vortex which entrain of part of the buoyancy interface resulting in billow-like structures. The entrainment is one-directional - lighter fluid gets warped into the billow. The primary vortex eventually produces some ejection events later.

Research Question: How does varying the perturbation amplitude a_p affect the onset and development of the asymmetric Holmboe instability in cold water?

Background Perturbation

- To initialize an instability, the initial velocity field is perturbed with some perturbation amplitude a_p. This perturbation leads to formation of waves of several wavelengths, with some of them growing faster than others and developing into a shear instability. The initial velocity field is given by,

$$u_i = u_0 \tanh\left(\frac{z - z_m}{L_U}\right) + a_p \sigma(0,1),$$

where $\sigma(0,1)$ is the Normal distribution with mean 0 and standard deviation 1.

- In nature, this initial perturbation can be of varying magnitude. Typically most simulations choose a single value somewhat arbitrarily which is large enough to result in an instability.
- The focus of this work is to understand how different magnitudes of initial perturbation

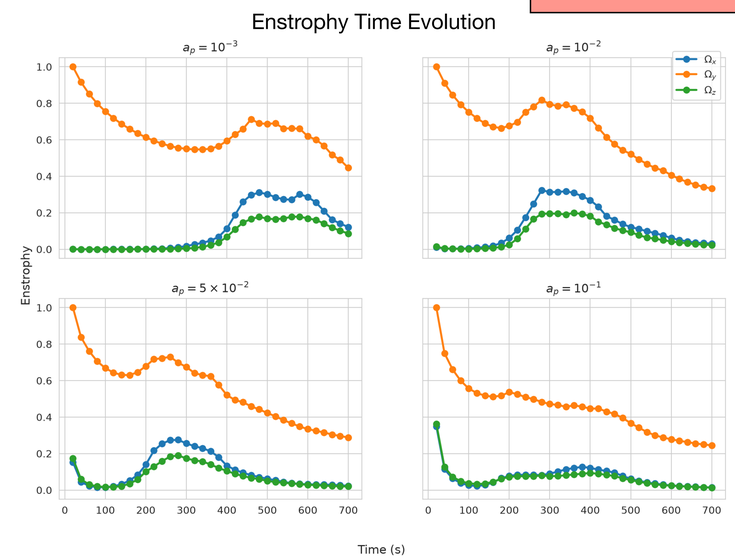
References

1. M. Stastna, K. Bhavsar, S. Hartharn-Evans and N. Castro-Folker; The three-dimensional state of stratified shear instability in cold water. *Physics of Fluids* 1 May 2025; 37 (5): 054112. https://doi.org/10.1063/5.0268302
2. K. Bhavsar, M. Stastna and N. Castro-Folker; Effects of weak buoyancy on shear instabilities in cold water. Physical Review Fluids 25 February 2026; 11 (2): 023904. https://doi.org/10.1103/11h6-qs7b
3. J. R. Carpenter, G. A. Lawrence, W. D. Smyth; Evolution and mixing of asymmetric Holmboe instabilities. Journal of Fluid Mechanics 10 July 2007; 582: 103–132. https://doi.org/10.1017/S0022112007005988

Perturbation Amplitude Dependence

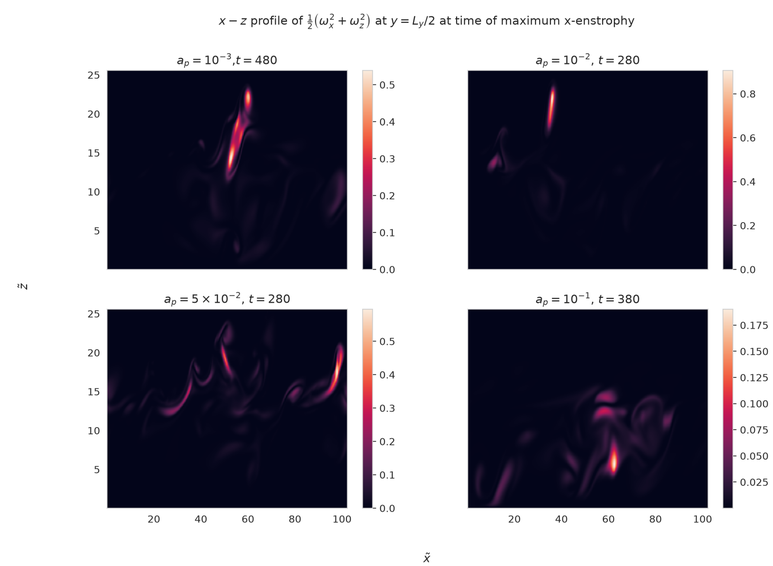
- We investigate the dependence of the dependence of the initial perturbation amplitude by studying the time evolution of the enstrophy (a measure of rotational energy) for four values of a_p:
  - $a_p = 10^{-3}$
  - $a_p = 10^{-2}$
  - $a_p = 5 \times 10^{-2}$
  - $a_p = 10^{-1}$

$\Omega_i = \frac{1}{2}\omega_i^2$, where $i \in \{x, y, z\}$ and ω_i is the $i-$ vorticity component.



The enstrophy components all develop differently in each case. For a low a_p (top-left), the enstrophy peaks much later than in cases where a_p is moderate (top-right and bottom-left). For the highest a_p (bottom-right), we observe a much more diminished peak suggesting that the vorticity dynamics are not as strong when the initial perturbation applied is high.

- For more details, we take a look at some slices of the enstrophy at times of maximum $x$and z components of enstrophy.



We notice two types of differences here. First, the concentration of large vorticity in a small region for lower values of a_p (top). In contrast, for higher a_p (bottom), the regions with high enstrophy are spread out. Second, while the maximum enstrophy in the first three cases is comparable, for $a_p = 10^{-1}$, there is a clear drop in the enstrophy values.