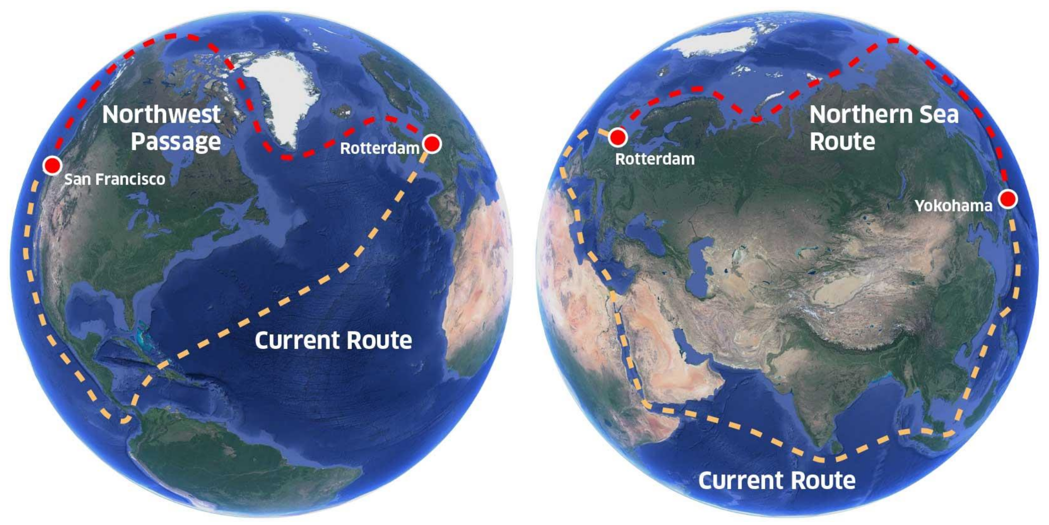
Integrating Nudging and Neural Operators in a Data-Driven Assimilation Framework

Maksym Veremchuk[1], Zhao Pan[1], Andrea K. Scott[1]
[1]Mechanical and Mechatronics Engineering, University of Waterloo

Global warming is rapidly reducing Arctic summer sea-ice extent, leading to increasing possibilities for Arctic shipping. Safe navigation under harsh Arctic conditions requires accurate short-term forecasts of sea-ice motion, which in part depend on reliable ocean current predictions. We present a hybrid data-assimilation framework that integrates a Fourier Neural Operator (FNO) with a nudging correction step: at each time step, the FNO forecasts the flow field, then a small nudging term pulls the forecast toward the latest observations, maintaining physical consistency. We tested our method on data from a quasi-geostrophic model, a simplified representation of large-scale ocean circulation. In benchmark experiments, our FNO-nudging system shows only a 4 % loss in accuracy compared to classical solvers but runs orders of magnitude faster, making real-time sea-ice trajectory forecasting and thus safer Arctic navigation, practically achievable.

Motivation



The new Arctic sea routes [1]

As new Arctic routes open, navigating through harsh ice conditions demands reliable forecasts of ice movement to adjust ship trajectories. To address this problem, we propose a **data-assimilation** framework that fuses observational data (e.g., satellite measurements) with model forecasts. Traditional physical solvers for ocean dynamics are too slow for real-time use, so we replace them with a **Fourier neural operator -** a fast, data-driven neural network that learns flow patterns. This hybrid approach delivers velocity estimates hundreds of times faster than classic solvers, enabling real-time predictions.

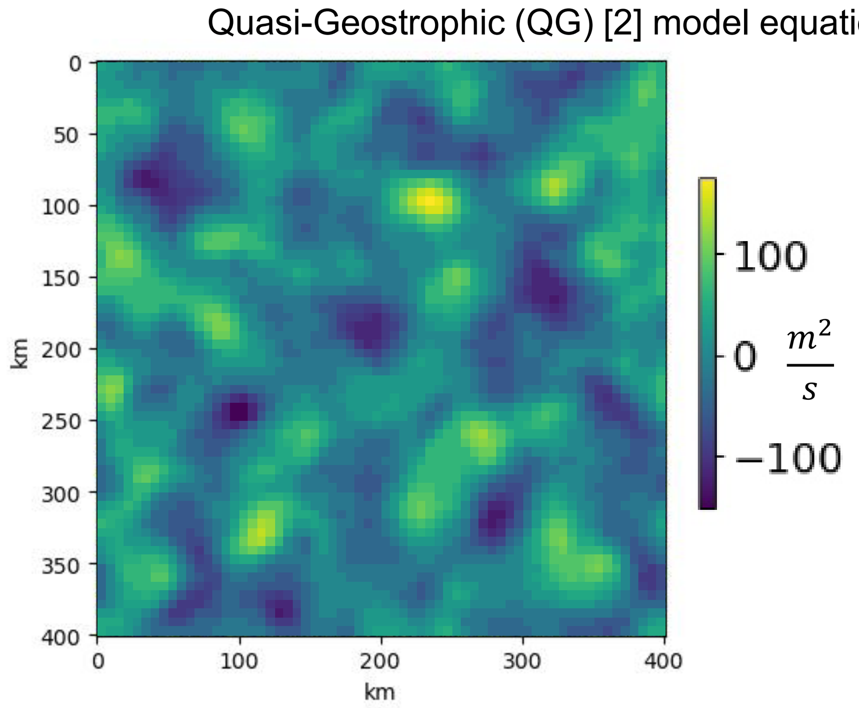
Ice floes in Arctic, example of an image taken from icebreaker "Henry Larsen" (Source: NRC)

Quasi-geostrophic model

$$\frac{\partial q_1}{\partial t} + \bar{u}_1 \frac{\partial q_1}{\partial x} + \frac{\partial \bar{q}_1}{\partial y}\frac{\partial \psi_1}{\partial x} + J(\psi_1, q_1) = ssd$$

$$\frac{\partial q_2}{\partial t} + \bar{u}_2 \frac{\partial q_2}{\partial x} + \frac{\partial \bar{q}_2}{\partial y}\frac{\partial \psi_2}{\partial x} + J(\psi_2, q_2) = -R_2\nabla^2\psi_2 + ssd$$

Quasi-Geostrophic (QG) [2] model equations to emulate the ocean dynamics

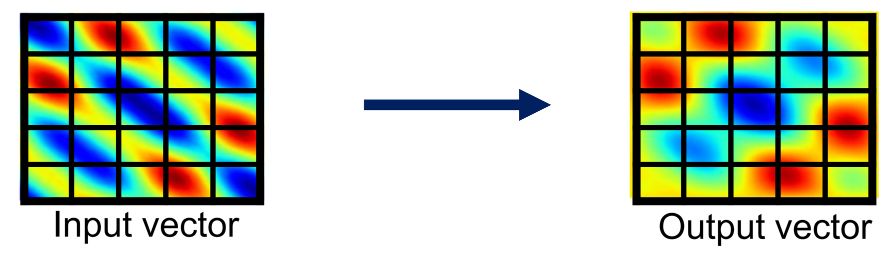


In the QG model, the stream function ψ captures the large-scale flow, and we obtain the ocean velocities by taking spatial derivatives of ψ. Because velocities drive ice motion, we assimilate the stream function directly into our framework - its use makes extracting and updating velocity fields straightforward.

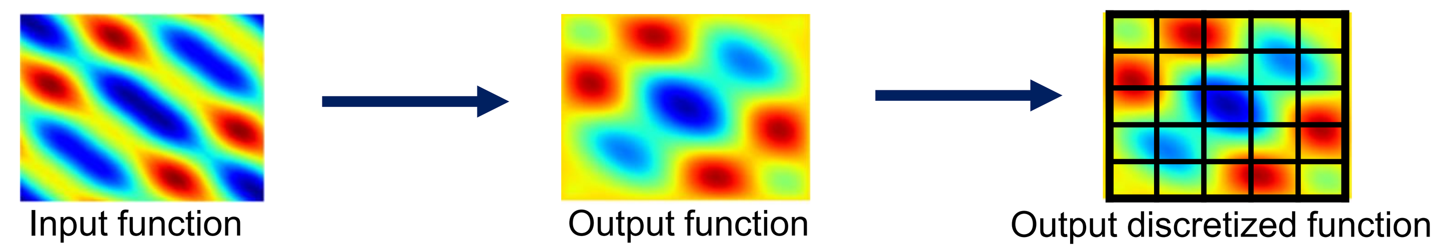
Fourier neural operator

A **Fourier Neural Operator (FNO)** [3] is a deep learning architecture that learns the underlying mapping between flow fields in the frequency domain, enabling rapid, data-driven predictions and seamless generalization across resolutions, even when deployed at grid sizes different from those seen during training.

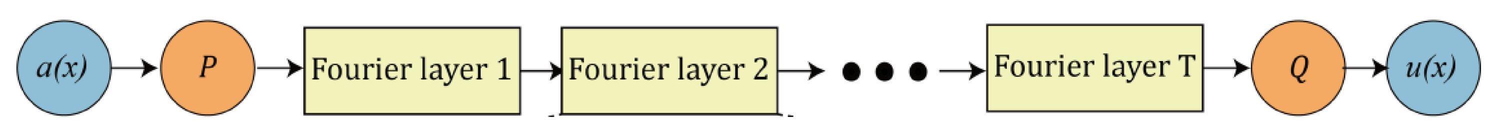
- Classical neural networks – discretize and learn:



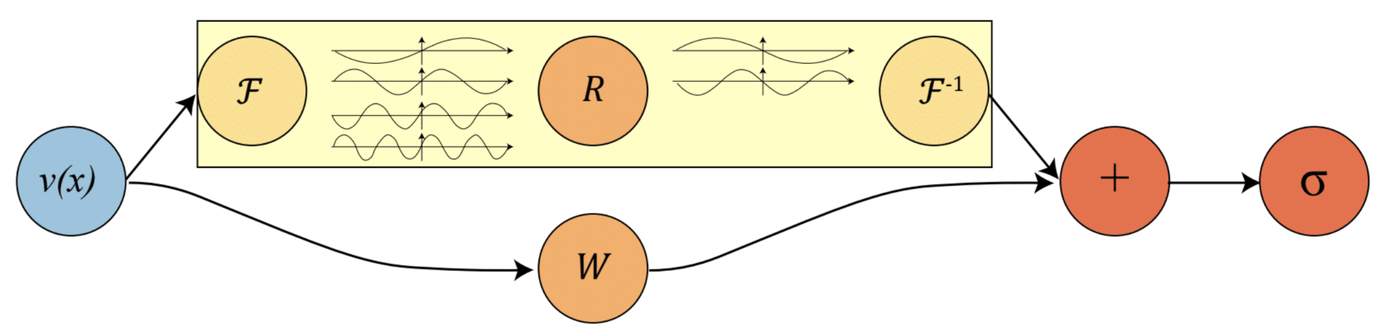
- Neural operator – learn and then discretize:



Architecture of Fourier neural operator:



FNO architecture



Fourier layer

Nudging data assimilation

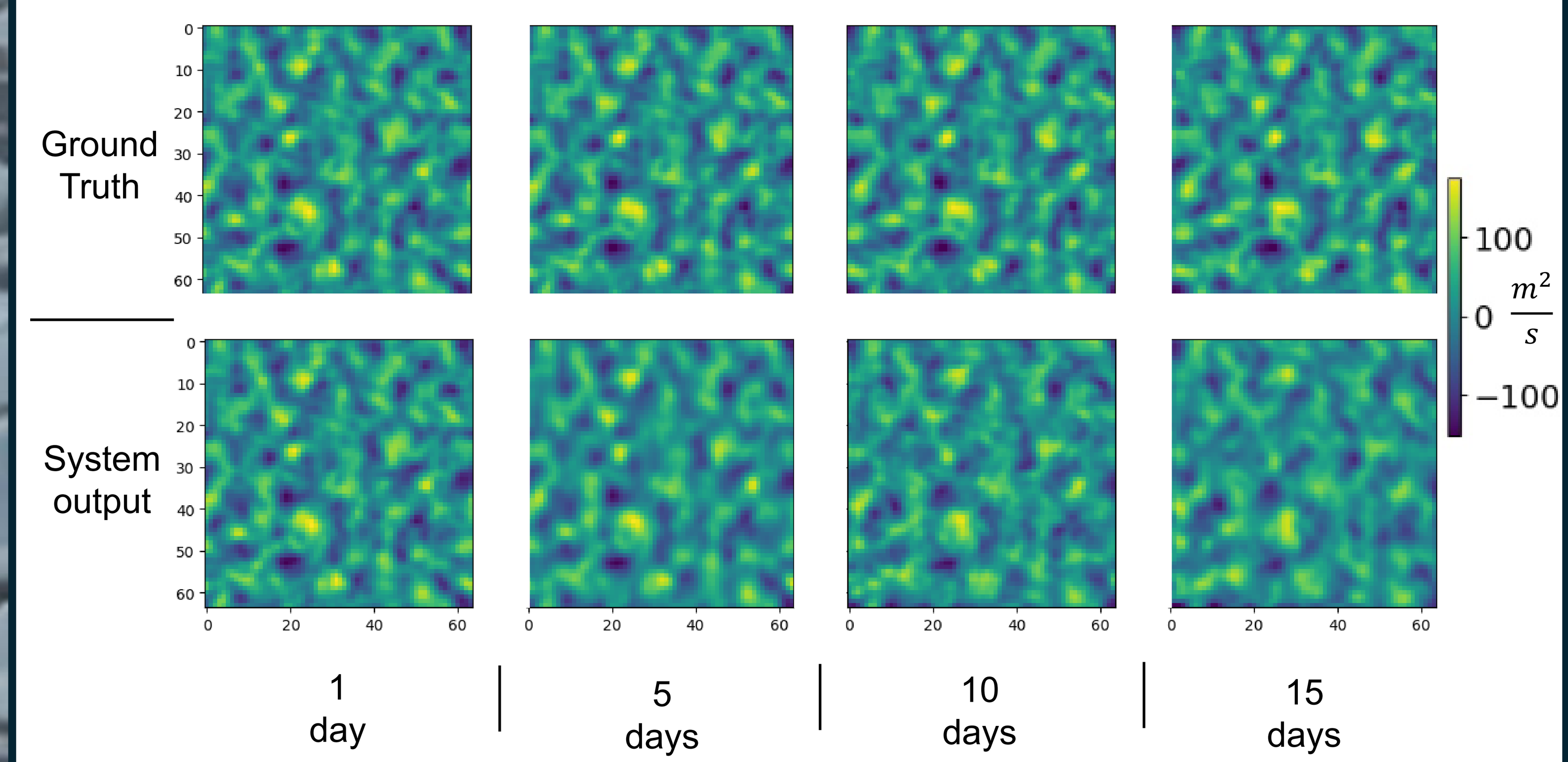
To fuse model forecasts and real-world measurements (x_{obs}), we use a **nudging data-assimilation** [4] to update the system state:

$$\dot{\hat{x}} = F(\hat{x}) - \kappa[I(\hat{x}) - I(x_{obs})],$$

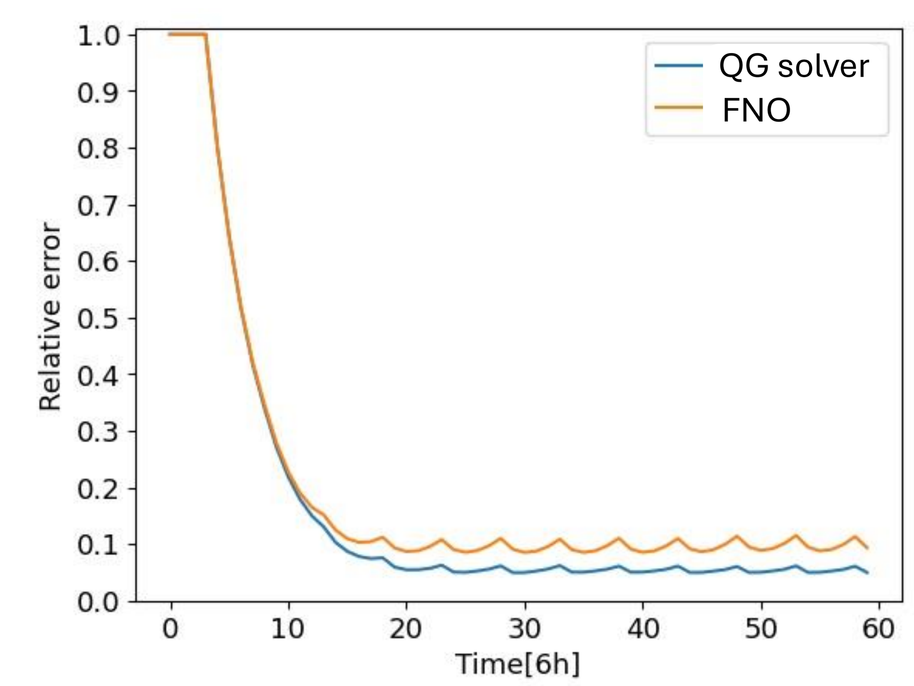
in each cycle, the model forecast $F(\hat{x})$ captures the system's dynamics ($\hat{x}$), then the term $\kappa[I(\hat{x}) - I(x_{obs})]$ corrects any mismatch with the real observations, where κ is a scalar relaxation parameter and I is interpolation operator (if data is not fully observed). While F can be implemented as a classical QG solver, we propose using a **Fourier Neural Operator** instead: it learns the underlying flow dynamics from data, delivers rapid inference, and seamlessly generalizes across grid resolutions, making real-time sea-ice trajectory forecasting both accurate and computationally efficient.

Results

Example of output for the proposed system:



Comparison of relative error for FNO and QG solver for 60 timesteps (360 hours):



Final system show speed up compared to physical solver (compared on 60 timesteps):

Domain size	Nudging FNO	Nudging GQ model
64 x 64	0.5 sec	1sec
512 x 512	5 sec	700 sec

References

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[2] J. Covington, N. Chen, and M. M. Wilhelmus, "Bridging Gaps in the Climate Observation Network: A Physics-Based Nonlinear Dynamical Interpolation of Lagrangian Ice Floe Measurements via Data-Driven Stochastic Models," *Journal of Advances in Modeling Earth Systems*, vol. 14, no. 9, Sep. 2022, DOI: https://doi.org/10.1029/2022ms003218.

[3] Z. Li *et al.*, "Fourier Neural Operator for Parametric Partial Differential Equations," *arXiv:2010.08895 [cs, math]*, Oct. 2020, Available: https://arxiv.org/abs/2010.08895

[4] A. Azouani, E. Olson, and E. S. Titi, "Continuous Data Assimilation Using General Interpolant Observables," *Journal of Nonlinear Science*, vol. 24, no. 2, pp. 277–304, Nov. 2013, DOI: https://doi.org/10.1007/s00332-013-9189-y.

Conclusion & future work

For FNO, accuracy degrades from 8% to 12% compared to GQ solver on tested trajectories but at the same time provides huge speed up (up to 700 times on large domains), which makes the developed data assimilation system a suitable choice for real-time navigation problems.

Future work: develop interpolation methods to handle missing or partial observations and seamlessly integrate them into our nudging data-assimilation framework.