Modelling Temperature Evolution Under Partial Ice Cover

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Background

- Partial ice cover could become more prevalent in northern lakes due to a warming climate
- Sunlight is mostly **absorbed** by open water and **reflected** by ice, so open water warms more easily than ice covered water
- Conversely, water **emits** much more heat than ice, so it loses heat more quickly
- These **nonuniform surface properties** can cause neighbouring zones of **ice cover** and **open water** to be different temperatures
- Due to water's nonlinear equation of state, this temperature difference can cause **mixing** between the two regions
- **Question**: Can we create a model of water temperature under partial ice cover that gives reasonable results while only using basic physics principles?

Model

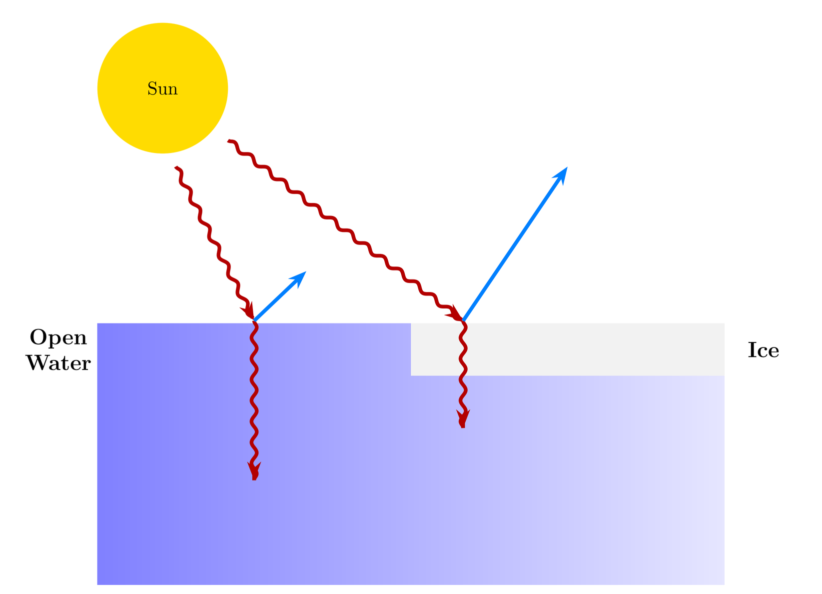
Incoming heat terms

$$q_{in} = \frac{Q}{4}(1-\alpha)$$

Albedo is the reflectivity of a surface

Incident Sunlight

Fraction Absorbed



Water: Low albedo Ice: High albedo

Emitted heat terms

$$q_{out} = \epsilon\ \sigma T^4$$

Stefan-Boltzmann law: Maximum possible heat any object can emit

Emissivity is the fraction of the maximum heat emission that a realistic object can achieve

Emission fraction is high for water and low for ice

Mixing terms

1. **Linear Heat Transport**

Follows Newton's law of cooling

$$k\Delta T$$

Convective heat transfer coefficient

Temperature difference between open water and ice-covered water

2. **Nonlinear Heat Transport**

$$k_3(\Delta T)^3$$

Nonlinear heat transfer coefficient

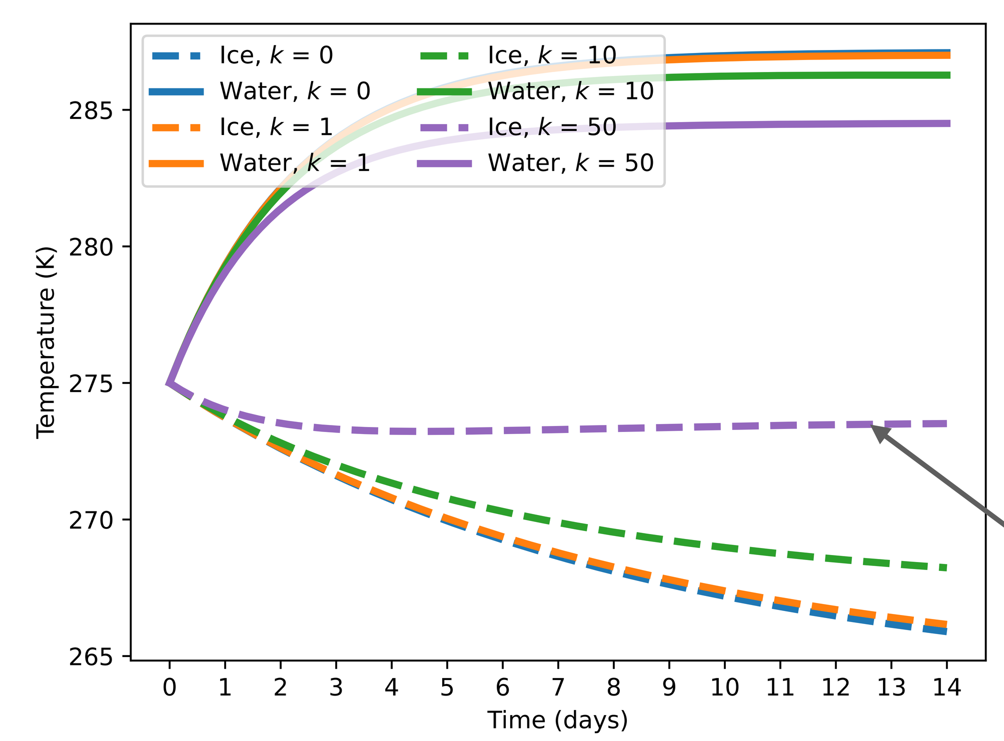
Nonlinear dependence drives more mixing

Results

1. Constant Insolation

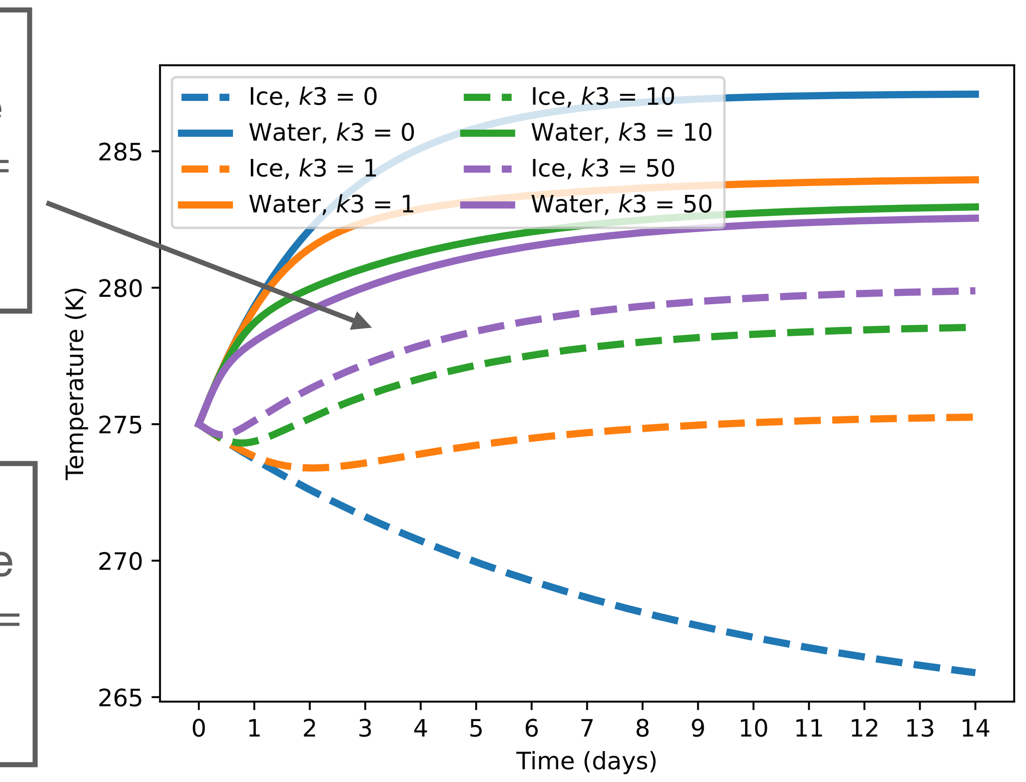
Model:

$$\begin{cases} \rho c_p V_1 \frac{dT_i}{dt} = A_{surface}(\frac{Q}{4}(1-\alpha_i) - \epsilon_i \sigma T_i^4) - A_{contact}(k(T_i - T_w) + k_3(T_i - T_w)^3) \\ \rho c_p V_2 \frac{dT_w}{dt} = A_{surface}(\frac{Q}{4}(1-\alpha_w) - \epsilon_w \sigma T_w^4) - A_{contact}(k(T_w - T_i) + k_3(T_w - T_i)^3) \end{cases}$$



Small temperature differences = strong mixing!

Large temperature differences = weak mixing!



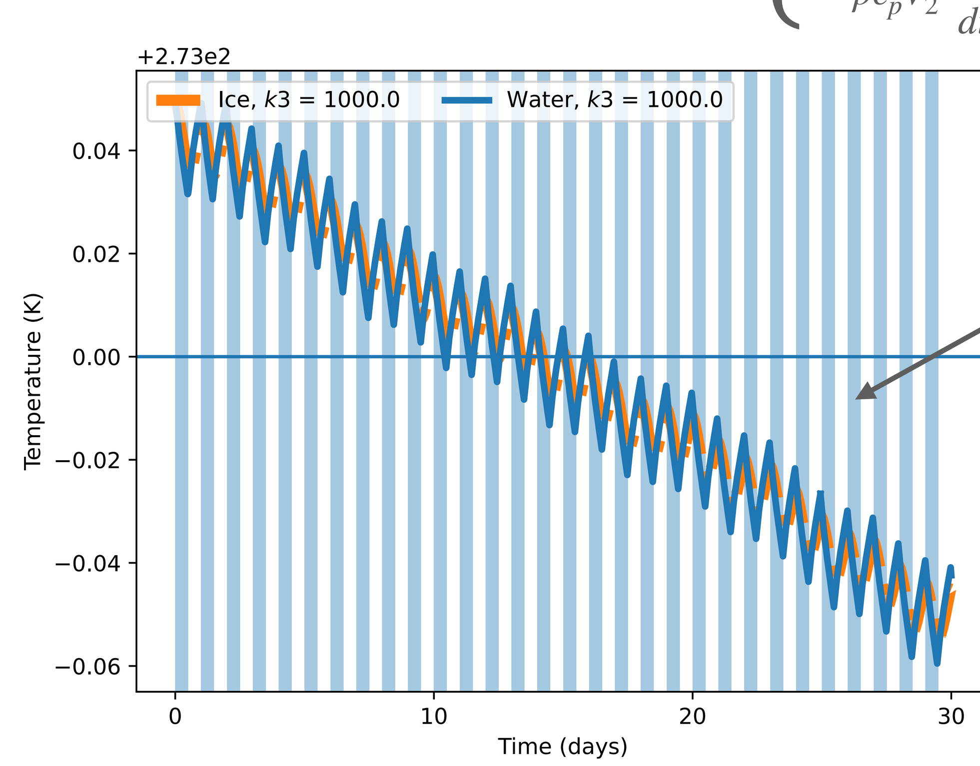
Results for **only linear heat transport**: not much mixing for small values of k!

Results for **only nonlinear heat transport**: strong mixing with very small values of k3!

2. Day-Night Cycle

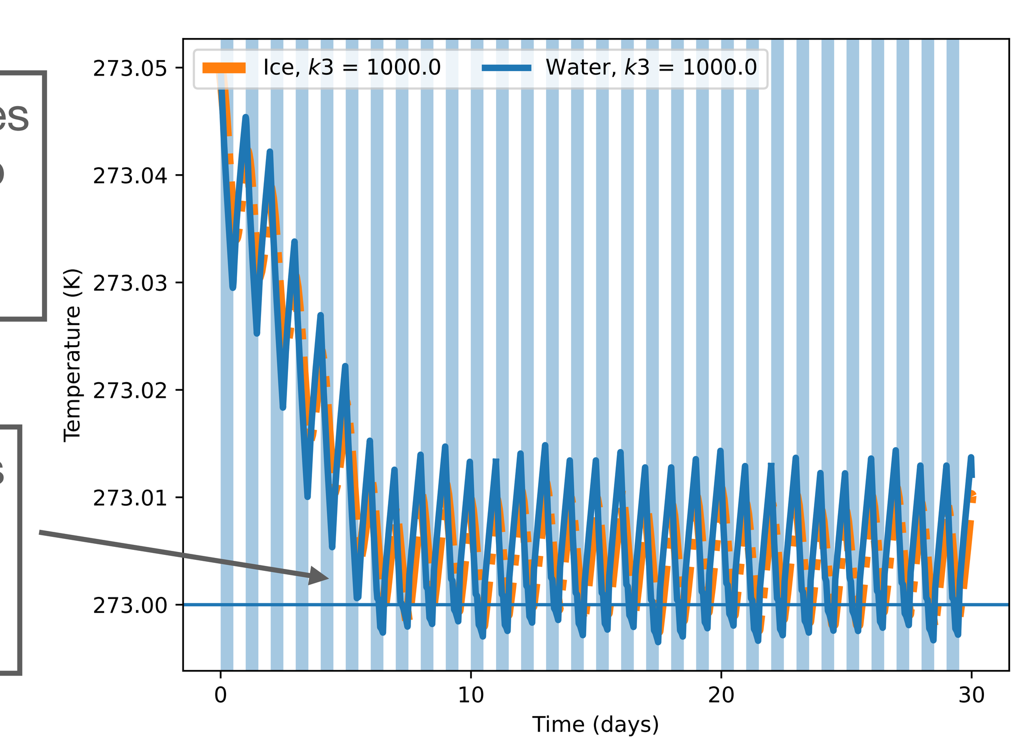
Model:

$$\begin{cases} \rho c_p V_1 \frac{dT_i}{dt} = A_{surface}(\frac{Q(t)}{4}(1-\alpha_i) - \epsilon_i \sigma T_i^4) - A_{contact}(k(T_i - T_w) + k_3(T_i - T_w)^3) \\ \rho c_p V_2 \frac{dT_w}{dt} = A_{surface}(\frac{Q(t)}{4}(1-\alpha_w) - \epsilon_w \sigma T_w^4) - A_{contact}(k(T_w - T_i) + k_3(T_w - T_i)^3) \end{cases}$$



Temperatures continue to drop past freezing!

Temperatures settle just above freezing!



Without emissivity switch **With emissivity switch**

When water drops below the freezing level, we switch its emissivity to be equal to the emissivity of ice!

Conclusions

- Can achieve **reasonable results** for temperature under partial ice cover with a **simple model**
- **Nonlinear** heat transport may be necessary to integrate **strong mixing** into the model
- Day cycle results show a need for **emissivity** values to reflect **state changes** (i.e., water freezing)